Mathematical Analysis - List 8

1. Each limit represents the derivative of some function f at some number x_0. State f and x_0 in each case.

a) $\lim\limits_{h\to 0}\dfrac{\sqrt{1+h}-1}{h}$; b) $\lim\limits_{t\to 0}\dfrac{(2+t)^3-8}{t}$; c) $\lim\limits_{x\to -1}\dfrac{x^9+1}{x+1}$;

d) $\lim\limits_{x\to 3\pi}\dfrac{\cos x+1}{x-3\pi}$; e) $\lim\limits_{x\to \pi}\dfrac{e^{\sin x}-1}{x-\pi}$; f) $\lim\limits_{t\to 2}\dfrac{\frac{t}{t^2-1}-\frac{2}{3}}{t-2}$.

2. Use the definition of a derivative to find $f'(x)$:

a) $f(x)=\dfrac{1}{x+1}$, if $x\neq -1$; b) $f(x)=\sqrt{3x}$, if $x>0$;

c) $f(x)=\operatorname{tg} x$, if $x\neq \dfrac{\pi}{2}+k\pi$ for $k\in\mathbb{Z}$; e) $f(x)=x^2-3x$, for $x\in\mathbb{R}$.

3. The position function of a particle is given by $s(t)=t^3-4.5t^2-7t$, $t\geqslant 0$. When does the particle reach a velocity of 5m/s?

4. If the tangent line to $y=f(x)$ at $(4,3)$ passes through the point $(0,2)$, find $f(4)$ and $f'(4)$.

5. Find an equation of the tangent line to the curve $y=\dfrac{x}{1+2x}$ at the point $(-\frac{1}{4},-\frac{1}{2})$.

6. Sketch the graph of a function f for which $f(0)=1$, $f'(0)=3$, $f'(2)=0$, and $f'(4)=-2$.

7. Use some rules of differentiation to find $f'(x)$.

a) $f(x)=\dfrac{x^2+1}{x^3+x}$; b) $f(x)=\dfrac{\sin x}{x^4+4}$;

c) $f(x)=\left(1+\sqrt[4]{x}\right)\operatorname{tg}\sqrt{x}$; d) $f(x)=\sin^6 x+\cos^6 x$;

e) $f(x)=\sqrt{\sin\dfrac{1}{x^4}+3}$; f) $f(x)=\cos\sqrt[3]{\operatorname{ctg}\left(x^2\right)}$.

8. Differentiate the function.

a) $f(x)=\ln(e^{-x}+xe^x)$; b) $f(x)=\ln(x+\ln x)$; c) $f(x)=\ln(x+\sqrt{x^2-1})$;

d) $f(x)=(\arcsin x)^2$; e) $f(x)=\sqrt{1-x^2}\arcsin x$; f) $f(x)=\arctan(x-\sqrt{x^2+1})$;

g) $f(x)=x^{1/x}$; h) $f(x)=(\sin 3x)^x$; i) $f(x)=x^{\ln x}$.

9. Use the definition of derivative to prove that $\lim\limits_{x\to 0}\dfrac{\ln(1+x)}{x}=1$.

10. Find f', f'', f'''.

a) $f(x)=x^3-\dfrac{2}{x}$; b) $f(x)=x\sin x$;

c) $f(x)=4x^7-5x^3+2x$; d) $f(x)=\sin^3 x+\cos^3 x$.

11. Find $f'_+(x_0)$ and $f'_-(x_0)$ and determine whether f is differentiable at x_0.

a) $f(x)=\left|x^2-x\right|$, $x_0=1$; b) $f(x)=\sin x\cdot \operatorname{sign}(x)$, $x_0=0$;

c) $f(x)=\left|\operatorname{ctg}^3 x\right|$, $x_0=\dfrac{\pi}{2}$; d) $f(x)=\left|x^5\right|$, $x_0=0$.